

Faceted Crystal Shape Evolution During Dissolution or Growth

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A model for the prediction of faceted crystal shape evolution during growth or dissolution is presented. An ab initio mechanistic model for the relative growth or dissolution rates is also described for organic molecular crystal systems. The shape evolution model proves that while growing crystals evolve toward a steady-state shape, dissolving crystals evolve away from a steady-state shape. Thus, crystals cannot achieve a steady-state during dissolution. This methodology may be used to probe crystal shapes that are obtainable by growing and/or dissolving crystals, including pharmaceutical crystals, inorganic materials and geological minerals. The technique has been successfully applied to predict the shape evolution of succinic acid growing or dissolving in water. © 2007 American Institute of Chemical Engineers AIChE J, 53: 1337–1348, 2007
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Introduction

In recent years, a major focus of chemical engineering has turned to microstructured products and materials, both crystalline and amorphous, such as those found in the food, pharmaceutical and electronics industries. This focus is derived from the unique performance and characteristics specific to the crystalline lattice structure. Solid structure is important in both two-dimensions, crystal surface structure, and in three dimensions, the crystal polymorph. Crystal dissolution is useful for engineering which surface structures are present, as well as for understanding and modeling systems where growth and dissolution occur simultaneously, such as Ostwald ripening or polymorph interconversion.

Surface properties of crystals depend upon which crystal faces are exposed and their relative sizes; therefore, the crystal shape is paramount in engineering the desired surface properties. One case where surface properties and morphology are important is in catalysis, where different index faces possess different catalytic activity. Steady-state and dynamic crystal morphology also affect down stream processes such as filtering, washing and drying, as well as particle flowability

and agglomeration. Thus, the ability to understand and manipulate crystal shapes enables both product and process improvements.

Dissolution is a valuable mechanism for exposing crystal planes and generating crystal shapes that are not easily obtained through growth alone. Extensive research has been performed for dissolution at high undersaturations (for example, in pure solvents), such as those in chemical etching¹ or pharmaceutical efficacy;² however, less attention has been directed towards dissolution at low-undersaturations. The dissolution mechanisms in these two cases are quite different, leading to different shapes and properties.

Dissolution at low undersaturations is also important in the context of polymorphism. Different crystal polymorphs have different crystal structures. As a result, different polymorphs of the same substance usually have different physical and chemical properties, such as solubility, melting point, color, bioavailability and compressibility. Thus, the ability to process and market the correct polymorphic product is vital. In order to understand the phenomenon of polymorphism, both the formation of polymorph, as well as their interconversion must be understood. Polymorph formation is determined by nucleation;³ whereas their transformation often specifically involves dissolution.^{4,5}

Industrial crystallizations are frequently carried out in solution. When a solution is present, polymorphs often preferentially

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transform through a solution mediated mechanism.⁶ A solution mediated-phase transformation, in a constant temperature-batch crystallizer, proceeds as follows. First, the metastable form grows until the solution composition is close to the solubility of this form. When the saturation concentration of the metastable form is reached it will stop growing. The stable form may nucleate at any point, determined by relative nucleation kinetics, before or after the saturation of the metastable form is reached. Then the stable form will grow, causing the solution to become undersaturated with respect to the metastable form, causing it to dissolve. Once the metastable form has completely dissolved at the expense of the growing stable form, the stable form will grow until the solution reaches its solubility with respect to the stable form.⁴ In order to fully understand this solution mediated polymorphic phase transformation it is necessary to be able to model both growth *and* dissolution of crystals. A characteristic feature of this process is that dissolution occurs at relatively low-levels of undersaturation.

The shapes of dissolved crystals have been previously discussed in the context of mineralogy, crystallography and thermodynamics. Moore suggests that the shapes of mineralogical crystals result from faces becoming vertices and vertices becoming faces.⁷ Frank proposes that crystals will disappear before changes in shape have ceased,⁸ and Gibbs⁹ (page 326) wrote, "The effect of dissolving a crystal (even when it is done as slowly as possible), is, therefore, to produce a form which probably differs from that of theoretical equilibrium in a direction opposite to that of a growing crystal." We demonstrate that in order to understand the effects of dissolution on crystal shape, a model for the shape evolution in dissolution is required. The key difference between morphology evolution during growth and dissolution is that growth shapes tend to be dominated by slow growing planes, while dissolution shapes are dominated by fast moving planes.

We have developed a generalized model that provides crystal shape evolution predictions for both growth and dissolution. This model provides significant insight into obtainable crystal morphologies, as well as conclusively demonstrating that crystals do not obtain a steady-state in dissolution (whereas they do in growth). The model for shape evolution in growth and dissolution, in combination with the mechanisms for the appearance and disappearance of faces, can be applied to any crystal system. We also describe a method to predict the relative dissolution rates of flat faces for molecular organic crystals at low undersaturations. The relative velocities of stepped and kinked faces are estimated based on diffusion limitations. The relative rates are predicted from a mechanistic model based on ideas first proposed by Burton et al.¹⁰ and on the physical principles proposed by Winn and Doherty.¹¹ The predicted relative dissolution rates are combined with the shape evolution model providing for a fully predictive model for crystal shapes evolving during dissolution at low-undersaturations.

This article is organized as follows. We begin by describing the relevant mechanisms for growth and dissolution at low-undersaturations. Then we provide methods for the prediction of these rates using mechanistic models. Next, we describe the shape evolution model, including methods for face selection based upon mechanisms for the appearance and disappearance of crystal faces. Finally, we provide two

examples. First, an illustrative crystal system is used to demonstrate the general utility of the model for any crystal system, organic or inorganic, where the crystal face dissolution rates could be determined by any desired measurement or modeling technique. Second, the dissolution of succinic acid in water is modeled using relative dissolution rates predicted from the models presented here.

Mechanisms of Growth and Dissolution

The mechanism for growth and dissolution of crystals varies depending on the classification of the crystal face.¹² Flat (F) faces, those containing more than one strong periodic bond chain running along the plane, grow or dissolve by the flow of steps across the surface, which are created either by a 2-D nucleation or a spiral mechanism (see Lasaga and Lutge¹³ for a discussion in the context of dissolution). The crystallization classification of a face being "flat" implies only the nature of the bond chain structure in the face. The surface of these faces will not be perfectly flat on a molecular level. Stepped (S) faces, those containing one and only one strong bond chain running along the plane, grow or dissolve by the evolution of every row of steps in the corrugated step surface. Finally, kinked (K) faces, those which contain no strong bond chains along their planes, grow or dissolve by the rapid incorporation or disincorporation of molecules from virtually any position on the crystal face, since the entire surface is covered by kinks that allow for such growth or dissolution. These kinked faces are inherently roughened since they do not possess any energetic barrier for the incorporation or disincorporation of crystal material, and their motion is normally governed by heat transfer or diffusion limitations. For a comprehensive discussion of the classification of crystal faces, see Hartman.^{12,14}

Flat crystal faces grow or dissolve by the flow of steps across a crystal surface. For growth, this flow of steps occurs by incorporation of the solute material from solution onto the existing crystal. This process is thought to occur in four stages. First, the solute diffuses from the bulk of the solution (stage 1) through a boundary layer to the crystal surface. The solute molecule then adsorbs onto the surface (stage 2), before diffusing across it (stage 3), until it is incorporated into the lattice (stage 4). For dissolution the same process occurs in reverse, with the molecule first disincorporating from the lattice, then diffusing across the surface, desorbing from the surface and finally diffusing through a boundary layer into the bulk solution. The most important incorporation or disincorporation location on the lattice for both growth and dissolution is the kink site. Such kink sites have been experimentally visualized in protein crystals.¹⁵

The growth/dissolution mechanism on flat faces is thus a function of the method by which kinked steps are created (see Figure 1). At low-super/under saturations flat faces will grow/dissolve via the spiral dislocation mechanism, where a continuous source of steps evolves from screw dislocations, which are in abundance on the surfaces of most real organic crystals.¹⁰ Note that a dissolution spiral can best be thought of as a growing pit rather than a dissolving hill. In recent years these spirals have been experimentally measured using AFM.¹⁶⁻¹⁹ Alternatively, 2-D nuclei can be the source of steps on a crystal surface. These are generally not easily

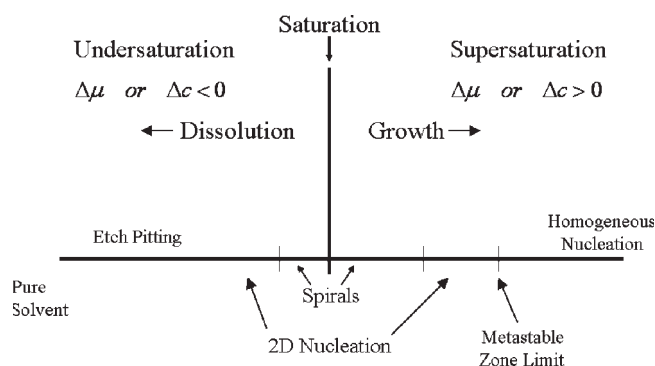


Figure 1. Relevant growth and dissolution mechanisms over the full range of super/undersaturations.

Note that at high supersaturations in growth the crystallization is dominated by 3-D homogeneous nucleation of new crystals, while in dissolution at low-undersaturations there is no direct equivalent, but etch pitting of the crystals occurs. Additionally, the relative chemical potential difference for a change to the 2-D nucleation mechanism in growth is at a larger value than the corresponding change to the 2-D nucleation mechanism in dissolution. This is due to the strain released by the preferential formation of nuclei on dislocations in dissolution (see Appendix).

formed at low-super/under saturations; however, at higher-super/under saturations they are often the primary source of steps.²⁰ Additionally, in dissolution these nuclei preferentially form on dislocations because that relieves additional (strain) energy from the system (See Appendix). A final source of steps in dissolution are the edges and vertices of the crystal. The dissolution of a crystal edge or vertex does not contribute to the relative dissolution rate of that crystal face, rather it will be a systematic source of a new face being formed. This will be further discussed in the context of shape evolution. At even higher levels of supersaturation, 3-D nucleation of new crystals dominates the crystallization, whereas in dissolution no equivalent phenomenon exists. At higher levels of undersaturation, the 2-D nucleation mechanism results in etch pitting.

Stepped crystal faces also grow and dissolve by molecules incorporating into steps; however, in this case, the faces themselves are corrugated and already populated by a ready source of steps. Rather than the steps flowing across the surface, each step grows or dissolves directly, leading to a perpendicular growth or dissolution rate. Thus, on such faces, no dislocations or 2-D nuclei need to be present for growth or dissolution. Along these steps, the kink sites will again be the most likely location for the incorporation or disincorporation of molecules.

Finally, on kinked crystal faces no steps are even necessary for kink sites to be formed. For kinked crystal surfaces, the entire surface is populated by kinks which allow for the ready incorporation (or disincorporation) of molecules.

Modeling Growth and Dissolution

Flat faces

The flow of steps in the form of a spiral about a screw dislocation was first proposed by Frank,²¹ and more fully developed in the landmark paper of Burton, Cabrera and Frank.¹⁰ The first application to dissolution came later by Cabrera and

Levine.²² The BCF expression for the rate of growth or dissolution G_{hkl} , normal to a crystal surface (hkl) is

$$G_{hkl} = \left(\frac{vh}{y} \right)_{hkl} \quad (1)$$

where v is the step velocity as it propagates across the surface, h is the step height, and y is the interstep distance. Since distances are measured as positive in the outward normal direction from an origin inside the body of the crystal, the expression is equally valid for the rate of dissolution normal to a crystal surface (hkl). The rate will be negative (implying a dissolving rather than a growing surface) as a result of the step height being negative for a dissolution spiral. An equivalent expression is

$$G_{hkl} = \left(\frac{h}{\tau} \right)_{hkl} \quad (2)$$

where τ is the characteristic rotation time of the spiral. This characteristic time is the time required to create one full turn of the spiral. It is also the amount of time between consecutive passes made by a step across a given location on the crystal surface. The characteristic time for the creation of one full turn of the spiral is

$$\tau_{hkl} = \left(\sum_{i=1}^n \frac{l_{c,i+1} \sin(\alpha_{i,i+1})}{v_i} \right)_{hkl} \quad (3)$$

where τ is summed over the n sides of the spiral, v_i is the step velocity of the i^{th} edge on face (hkl), and α is the angle between sides of the spiral. This assumes a step velocity profile consistent with Voronkov²³ which can be expressed as

$$\begin{aligned} v &= 0 & l \leq l_c \\ v &= v_\infty & l > l_c \end{aligned} \quad (4)$$

The critical length, l_c , is taken to be the critical size for the formation of a 2-D nucleus of the same shape formed by the spiral.

Modeling the relative velocity of a growth or dissolution step are completely equivalent; however, isotropic features of an absolute growth or dissolution rate expression, such as the entropy contribution to the free energy barrier are different. When kink integration limits the step velocity on a flat face, which often holds for solution crystallization,²⁴ the step velocity is directly proportional to the kink density.²⁰ Eliminating isotropic terms, the step velocity expression for the i^{th} edge on face (hkl) is given by

$$v_i \sim \frac{a_{p,i}}{1 + 0.5 \exp\left(\frac{\phi_{kink,i}}{kT}\right)} \quad (5)$$

where $a_{p,i}$ is the distance that the i^{th} edge has propagated by adding one growth unit. The remainder of the expression originates from the probability of finding a kink site at equilibrium, where $\phi_{kink,i}$ is the free energy required to create a kink along that edge in the presence of the solution.¹⁰ Methods have been developed to estimate this energetic interaction.²⁵

The critical length for the side of a spiral is normally estimated using a Gibbs-Thomson approach. Since one turn of

the spiral encompasses a dislocation, it is appropriate to include the energy associated with adding or removing a new layer of crystal to the dislocation in the analysis. However, this strain component to the energy is often small when compared to that of the energies associated with forming a new edge and adding or removing solute, and, thus, it will not be included here (see Appendix for a complete justification).

While the spiral mechanism is often the mode of growth or dissolution for flat faces at low-super/undersaturations, some cases exist where flat faces grow or dissolve solely by the 2-D nucleation mechanism. Modeling of the 2-D nucleation mechanism requires evaluating not only the spreading velocity of these nuclei, but also the rate at which they form. Such a detailed model of 2-D nucleation does not exist; however, approximate methods for predicting relative rates of crystal faces growing by 2-D nucleation have been developed.¹¹ These methods also have the potential for application in dissolution.

Stepped and kinked faces

Mechanistic models for relating the relative growth or dissolution rate of stepped and kinked faces to those of flat faces are less well developed. In the case of organic crystals growing or dissolving in solution, the rates of growth or dissolution of their stepped and kinked faces will be limited by diffusion. It has been proposed¹¹ that the growth rate of these higher index faces be estimated as an order of magnitude higher than the corresponding flat faces. This is also a good first approximation for the relative dissolution rates of these faces as well. In other crystallization systems (vapor growth/dissolution, inorganic materials, and so on.) where the relative magnitude of kink and bond energies provide for kinked faces to grow or dissolve even more rapidly than stepped faces, a velocity for the kinked faces should be estimated to be much higher than the corresponding stepped faces.

Evolution of Crystal Shape

Using the earlier modeling techniques to predict the relative growth or dissolution rates of crystal faces, in conjunction with a shape evolution model, the time-dependence of the crystal shape can be predicted. Such a model has recently been developed in the context of growth.²⁶ We have extended this model to include dissolution, revealing several novel conclusions which are independent of the method by which relative growth or dissolution rates are obtained (for example, from the earlier mechanistic model, experimental data or other modeling techniques.)

The model consists of a number of states equal to the number of planes that are actually present on the surface of the crystals (real planes), plus those that could appear on the surface of the crystal through growth or dissolution (virtual planes). The evolution of each real face is governed by an ordinary differential equation. Coupled to this set of differential equations is an algebraic condition that determines whether or not a virtual plane will appear on the crystal at the next time step. If a virtual plane does not appear, it is moved with the same velocity as the edge or vertex on which it is located until the next time step. First, we present the differential equations describing the shape evolution. Then, we describe

the method for determining the list of candidate planes. Finally, the condition for face appearance is explained.

Shape evolution

The simplest expression for a faceted crystal shape evolution model follows the perpendicular distance of each crystal face in time

$$\frac{dH_i}{dt} = G_i, \quad i = 1, \dots, N \quad (6)$$

where H_i and G_i are the perpendicular distance and normal velocity of face i , respectively, and N is the number of real crystal faces, which may change as the crystal evolves. This shape evolution equation is equally valid for both growth and dissolution; but in either case, it has no apparent steady-state. By appropriately nondimensionalizing the variables, steady-state features of the model can be revealed. From this point, the absolute and relative growth and dissolution rates will be distinguished by the subscripts G and D , respectively. Choosing the perpendicular distance of a reference face, H_{ref} , as the characteristic length, and growth rate of the same reference face, $G_{G,\text{ref}} = \frac{dH_{\text{ref}}}{dt}$, as the characteristic velocity; the dimensionless perpendicular distance of face i is $x_i = H_i/H_{\text{ref}}$, and the (dimensionless) relative growth velocity of face i is $R_{G,i} = G_{G,i}/G_{G,\text{ref}}$. Using these dimensionless variables we can reformulate the dynamic model as follows

$$\frac{dx_i}{dt} = \frac{G_{G,\text{ref}}}{H_{\text{ref}}} (R_{G,i} - x_i), \quad i = 1, \dots, N-1 \quad (7)$$

$$\frac{dH_{\text{ref}}}{dt} = G_{G,\text{ref}} \quad (8)$$

At each point in time the convex hull of the crystal reveals the crystal shape.

A dimensionless warped time then can be defined for growth, $d\xi_G = \frac{G_{G,\text{ref}}}{H_{\text{ref}}} dt$, and Eq. 7 can be rewritten in fully dimensionless form suitable for growth

$$\frac{dx_i}{d\xi_G} = R_{G,i} - x_i, \quad i = 1, \dots, N-1 \quad (9)$$

As time increases, the warped time also increases during growth since both the characteristic velocity ($G_{G,\text{ref}}$) and length (H_{ref}) are positive during growth. For dissolution the characteristic velocity is now the dissolution rate of the reference face, $G_{D,\text{ref}} = \frac{dH_{\text{ref}}}{dt}$. For the warped time to increase as the time increases in dissolution, the warped time must be redefined, $d\xi_D = -\frac{G_{D,\text{ref}}}{H_{\text{ref}}} dt$, since the characteristic velocity is now negative while the characteristic length is still positive. Eq. 7 then can be rewritten in a fully dimensionless form suitable for dissolution

$$\frac{dx_i}{d\xi_D} = x_i - R_{D,i}, \quad i = 1, \dots, N-1 \quad (10)$$

A steady-state of these systems corresponds to the condition when each of the states of the model, those corresponding to

both real and virtual faces, are at a value such that they do not change with time.

Under constant relative growth rate conditions, the growth form of the model is a linear system of ordinary differential equations, with all the eigenvalues equal to -1 ; therefore, it has the expected result of the steady-state being unique and stable. It is defined by

$$x_i^{ss} = R_{G,i}, \quad i = 1, \dots, N-1 \quad (11)$$

which is often expressed in the equivalent forms

$$\frac{R_{G,i}}{x_i^{ss}} = 1 \quad \text{or} \quad \frac{G_{G,i}^{ss}}{H_i^{ss}} = \frac{G_{G,\text{ref}}}{H_{\text{ref}}}, \quad i = 1, \dots, N-1 \quad (12)$$

Thus, the relative perpendicular distance of each face is equal to its relative growth rate, which is precisely the Chernov condition.²⁷ Since the crystal shape always evolves towards its unique stable steady-state, the evolution trajectories will depend on the initial condition; however, the final shape will always be identical. Also, disturbances in the system will be relatively unimportant since the crystals will again evolve toward the same steady-state shape after the disturbance is mitigated. The crystal shape will become more and more self-similar as the system approaches its steady-state. When the system is at the steady-state value, the crystal will evolve with self-similar shape. Under nonconstant relative growth conditions,²⁸ Eq. 9 must be coupled to the mass and energy balances that describe the evolution of the temperature and supersaturation fields (which determine the change in growth rate). In this case, the steady-state equations must be coupled to the steady-state mass and energy balances. The overall model (both dynamic and steady-state) is then nonlinear, with all the consequences that this implies.

Under constant dissolution conditions, the dissolution form of the model is also a linear system of ordinary differential equations; however, all of the eigenvalues are equal to $+1$. Thus, the unique steady-state of the system

$$x_i^{ss} = R_{D,i}, \quad i = 1, \dots, N-1 \quad (13)$$

is conclusively shown to be unstable. In fact, since the steady state is unstable, the system will constantly be evolving away from the steady-state. As a result, the initial condition will have an effect on the resulting evolution trajectory, as well as possible crystal end shapes. Additionally, disturbances will affect the shapes obtainable in dissolution. As the crystal dissolves and moves away from the steady-state, it will become less and less self-similar. (It is mathematically feasible for the crystal to obtain a self-similar shape in dissolution, by the disappearance of all, but a single set of planes at equal distances dissolving at equal rates; however, the crystal will not uniquely evolve toward that specific shape for any but the most special initial conditions nor will it maintain that self-similar shape except in the most perfect of dissolution conditions.) Under nonconstant dissolution conditions Eq. 10 must be coupled to the mass and energy balances, and the model is, like the growth model, also nonlinear.

Candidate face selection

In order to fully describe crystal shape evolution, a method to select all of the relevant planes for the simulation is neces-

sary. Since the slow growing planes are important in growth and there are relatively few of those, including all of them in a growth evolution simulation is a feasible, as well as correct selection. However, in dissolution, since the fast dissolving planes are important and there are an infinite number of these planes, selecting all of them is not feasible. Additionally, only specific planes will appear via specific molecular mechanisms during dissolution. Zhang et al.²⁶ demonstrated the possible macroscopic manifestations of new faces appearing and disappearing at edges and vertices during generalized shape changes specifically for growth. Here we present the molecular mechanisms that determine which specific faces can appear during dissolution. The candidate faces are then determined by an algorithm for selecting all of the planes necessary to perform a successful growth or dissolution shape prediction.

Face appearance at edges in dissolution

Flat faces are primarily bounded by edges containing a periodic bond chain, since the dominant faces of a grown crystal usually contain multiple bond chains. This allows for the crystal edges to be sources of steps in dissolution. In this case, the steps will be kinked in an identical fashion to a spiral edge and will act as a source of steps along both crystal faces with which it intersects. If the steps that emanate from a crystal edge are spaced identically to those on the spirals they will merely contribute to the layer-wise dissolution of the crystal faces. However, the actual spacing will be a distance of one growth unit since the removal of one row of growth units immediately allows for a second row to be exposed and then kinked. Thus, the edges can cause the formation of a new crystal face or faces rather than just contributing to the layer-wise dissolution of the adjacent crystal faces. Subsequently, as long as the face that is first exposed is not itself a flat (F) face (see section on Mechanisms of Growth and Dissolution for a description), only *one* new face will appear at this edge in dissolution since there is no driving force for any of the growth units on that face to be removed faster or slower than any others. We have neglected the effect that the bulk diffusion field would have on the possibility for some of the growth units along that edge to be removed faster than others, providing for additional faces to appear and hence rounding or smoothing to occur. Therefore, two cases need to be considered, corresponding to whether or not the initial face that could appear is an (F) face.

First, consider which crystal face initially appears along the edge between the (001) and (010) faces in Figure 2a. This face is the one that is exposed when the first row of growth units are removed, exposing two rows of growth units that will exist as the new face (Figure 2b). Thus, the new face will be the crystallographic plane that is precisely in the middle of the two previous planes that intersected at the edge. In this case, after those two new rows of growth units are exposed, they then become kinked and are removed (Figure 2c). This now leaves three rows of growth units that are exposed. If they do not compose an (F) crystal face then they all can be kinked and removed leaving four new rows of growth units, and so on (Figure 2d). This process continues providing for the macroscopic appearance of a single new crystal face (Figure 2e–f). Each of the rows of growth units are removed at the same speed since we have neglected

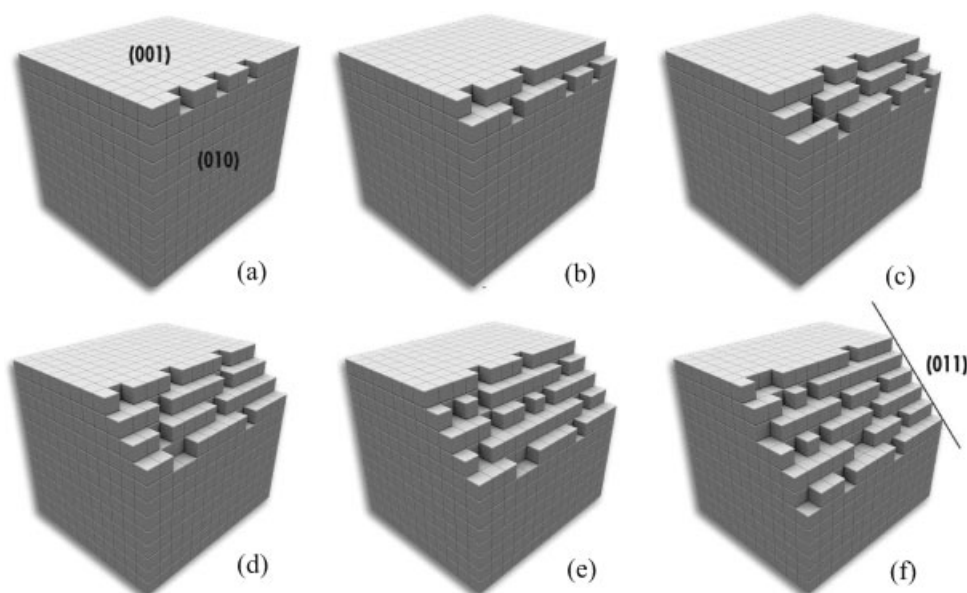


Figure 2. Dissolution at the edge of the (001) and (010) faces.

Since the first appearing face (011) is not a flat face, it is the only face that appears.

the influence of bulk diffusion, and they each have the same kink energy. The proper indexing of this additional crystal face is straightforward with the given knowledge of the growth unit and the unit cell.

If the three rows of growth units that were exposed (Figure 3d) comprised an (F) crystal face, then the middle row cannot be readily kinked, while the two adjacent rows are still present, since a bond connects these rows to one another. Thus, only the outer two rows can be kinked and removed, which now allows for the previous middle row to also be kinked along with the two newly exposed rows (Figure 3e). This process continues providing the possibility for exposing two new crystal faces. In other words, the intermediate flat

face that began to appear could not readily dissolve, causing the formation of a new face between it and each of the adjacent crystal faces (Figure 3f–h). Again, we have neglected the effect of the macroscopic diffusion field providing for an equal rate of removal of growth units limiting the smoothing of the edge that may occur. The indexing of these new crystal planes directly follows from the known information of the growth unit and the unit cell.

Face appearance by vertex dissolution

While the edge of a crystal first needs to be kinked in order for growth units to be removed, crystal vertices provide

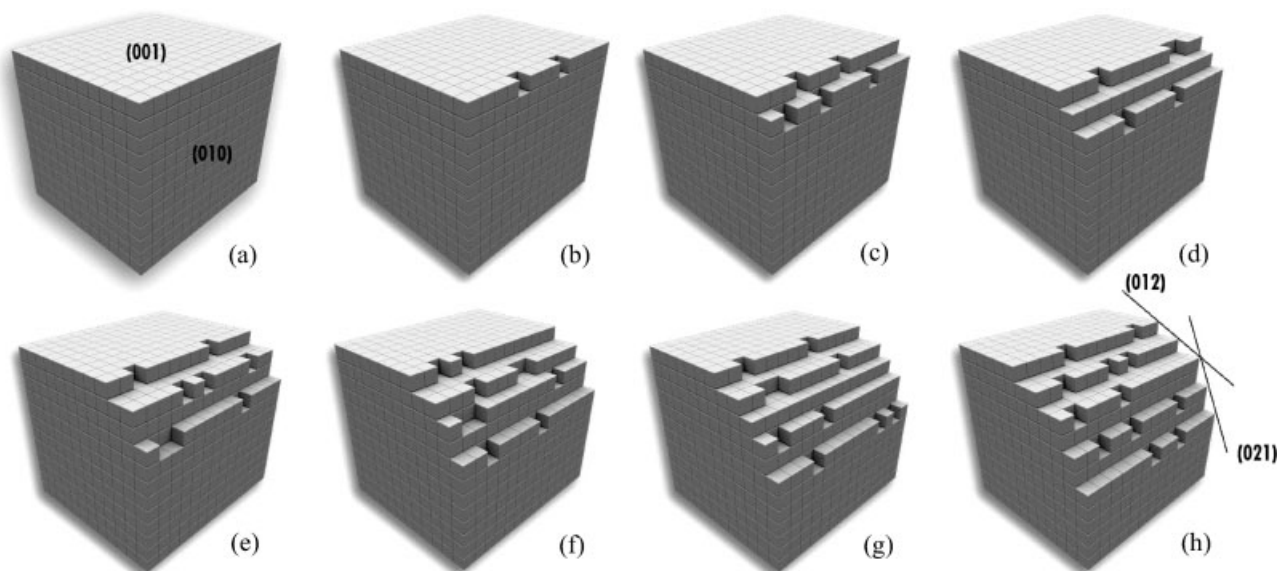


Figure 3. Dissolution at the edge of the (001) and (010) faces.

Since the first appearing face is a flat face (011), it does not appear macroscopically, rather two other faces ((021) and (012)) do appear.

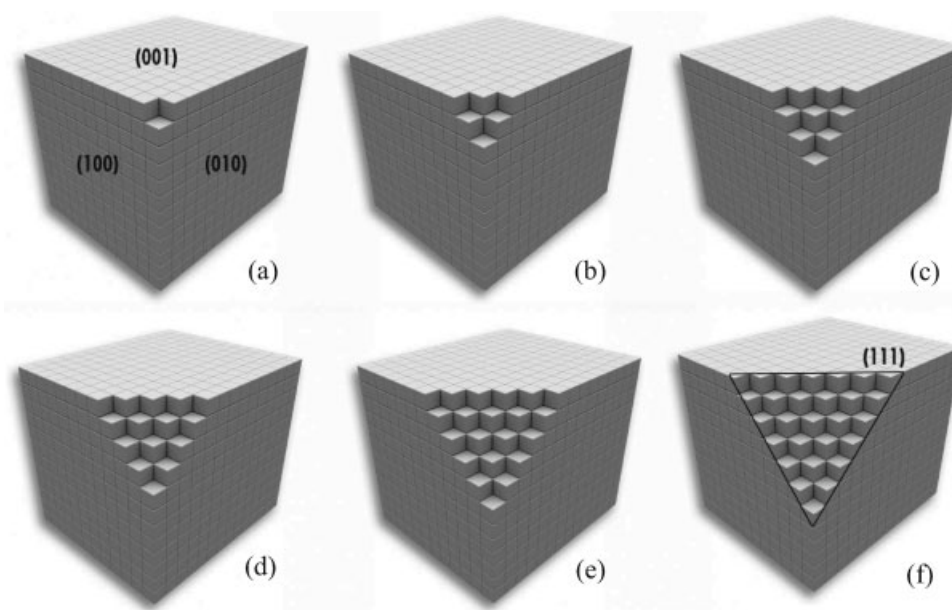


Figure 4. Dissolution at the vertex of the (100), (010), and (001) faces resulting in the appearance of a single plane (111).

for an already kinked molecular position. Because the growth unit at the vertex is in a kinked position it can be readily removed revealing a layer of growth units beneath (Figure 4a). This is an additional mechanism by which new faces can appear in dissolution. Similarly to faces appearing at edges, the face that is exposed in removing this molecule may or may not be an F face. If it is not, again only a single new crystal face could appear with the succession of molecules being removed, since all of the newly exposed molecules are equally likely to be removed. The indexing of this face is identical in determination to those new faces formed at the edges (Figure 4b–f). If the newly exposed face is an F face, additional planes will develop between it and the adjacent planes. Each of the possible combinations that could develop in this situation are not enumerated here; however, they directly follow from the method described earlier for edges.

Candidate face selection algorithm

In the previous sections, we have outlined the microscopic mechanisms for the appearance of new faces at edges and vertices. When describing a crystal shape evolution those new faces are just one of the three classes of faces that must be included. First, any candidate faces that are on the initial shape must be included in the superset. Second, each flat (F) face must be included. Finally, as described earlier, each face that exists at an edge or vertex formed by adjacent sets of those flat faces must also be included. The combination of these three types of faces provide for the total superset of candidate faces.

Test for face appearance

In order to fully describe crystal shape evolution, a method to predict the appearance and disappearance of faces is also required. Zhang et al.²⁶ determined the condition for face

appearance during growth. In growth, new faces appear when the magnitude of a virtual-face velocity is *below* a critical value, such that its appearance on the growth form is favorable. A similar condition exists for crystals in dissolution, where new faces appear when the magnitude of a virtual face velocity is *above* a critical value such that its appearance is favorable.

For each of the new faces that potentially appear at the edges and vertices, a macroscopic test for whether that face can appear must be considered. Consider an allowable virtual face that is not on the crystal F_v , with Miller index (h_v, k_v, l_v) , and normal growth or dissolution rate G_v located at the vertex P_{ijk} of three intersecting faces F_i , F_j and F_k , with growth or dissolution rates G_i , G_j and G_k . The spatial velocity vector v_{ijk} of this vertex point is

$$v_{ijk} = \begin{bmatrix} h_i & k_i & l_i \\ h_j & k_j & l_j \\ h_k & k_k & l_k \end{bmatrix}^{-1} \begin{bmatrix} G_i \\ G_j \\ G_k \end{bmatrix} \quad (14)$$

The projection of this velocity onto the normal direction of F_v is then given by

$$\rho_v = \frac{v_{ijk}^T \mathbf{n}_v}{\|\mathbf{n}_v\|} \quad (15)$$

where $\mathbf{r}_1^T \mathbf{r}_2$ represents the inner (dot) product between vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, $\|\mathbf{r}_1\|$ represents the norm of vector $\mathbf{r}_1$, and $\mathbf{n}_v$ represents the vector normal to face (h_v, k_v, l_v) . In general, the velocity vectors $G_v \mathbf{n}_v$ and v_{ijk} will not be parallel, but $G_v \mathbf{n}_v$ and $\rho_v \mathbf{n}_v$ are parallel, so their magnitudes (speeds) can be directly compared.

In growth, a face will appear if G_v is less than ρ_v . In this case face F_v is growing at a rate below the critical value ρ_v , for its appearance causing it to “grow in”. Otherwise, when

G_v is greater than ρ_v , face F_v will not grow in. In dissolution, a face will appear if $|G_v|$ is greater than $|\rho_v|$. In this case face F_v is dissolving at a rate above the critical value $|\rho_v|$, for its appearance causing it to “dissolve in”. Otherwise, when $|G_v|$ is less than $|\rho_v|$, face F_v will not be able to “dissolve in”.

Results

Here we present the results of implementing the evolution model in both growth and dissolution modes. First we present the results of an illustrative system for the case of dissolution where kinked and stepped faces dissolve at approximately the same rate (growth or dissolution of organics in solution). Then we present the results for the same illustrative crystal system where kinked faces dissolve much more rapidly than stepped faces (vapor growth/dissolution of organic or inorganic materials). These examples highlight the versatility of the model to cope with any system dissolving at low-undersaturations. Finally, we present the results of combining both the microscopic features of the mechanistic growth and dissolution rate predictions, for a small molecule organic solute in solution, in combination with the macroscopic features of the shape evolution methods. These results for succinic acid growing and dissolving in water further demonstrate the evolution towards a steady-state during growth in contrast to the evolution away from the steady-state in dissolution. The previously discussed features of face appearance under different initial conditions are also highlighted.

Example illustrative system

The illustrative system contains a single growth unit per unit cell with the unit cell parameters $a = 1 \text{ \AA}$, $b = 1 \text{ \AA}$, $c = 1 \text{ \AA}$, $\alpha = 90^\circ$, $\beta = 90^\circ$ and $\gamma = 90^\circ$. Two sets of peri-

odic bond chains exist in the system. One set of two periodic bond chains run along the $\langle 010 \rangle$ and $\langle 001 \rangle$ directions, while a second set of bond chains runs along the $\langle 100 \rangle$ direction. Each set of periodic bond chains has its own characteristic energy. Thus, the relative growth and dissolution rates of the $\{100\}$ plane family will be different than that of the $\{010\}$ and $\{001\}$ plane families. The relative growth and dissolution rates of the $\{100\}$ plane families are set as 2 and -2 , while the relative growth and dissolution rates of the $\{010\}$ and $\{001\}$ plane families are set as 1 and -1 . We use the steady-state growth shape as the initial condition for the dissolution evolutions, thus, the only additional candidate planes are those that are located at the edges and vertices of the adjacent flat faces. There are twelve planes at the edges of the adjacent flat planes corresponding to the $\{011\}$, $\{101\}$ and $\{110\}$ plane families. Additionally, there are eight planes at the vertices formed by adjacent sets of flat planes corresponding to the $\{111\}$ plane family.

First, we dissolve the crystal under the conditions which are appropriate for dissolution of an organic crystal in solution, where the kinked and stepped faces dissolve approximately equally rapidly, in this case with a dissolution rate of -10 . Figure 5a shows the steady-state growth shape, which is the starting shape for the dissolution evolution. Figure 5b–d shows the shape changes during the dissolution evolution. As the crystal dissolves it becomes more needle-like, and, as expected during dissolution, the shape becomes less self-similar. Since the dissolution rate of the virtual faces at the vertices is similar to that of the faces at the edges, intermediate planes only appear at the edges, but not at the vertices. Also, no more faces should be added to the candidate set since the molecules at the newly formed edges cannot be removed any more rapidly than those on the stepped faces that formed in the first instant.

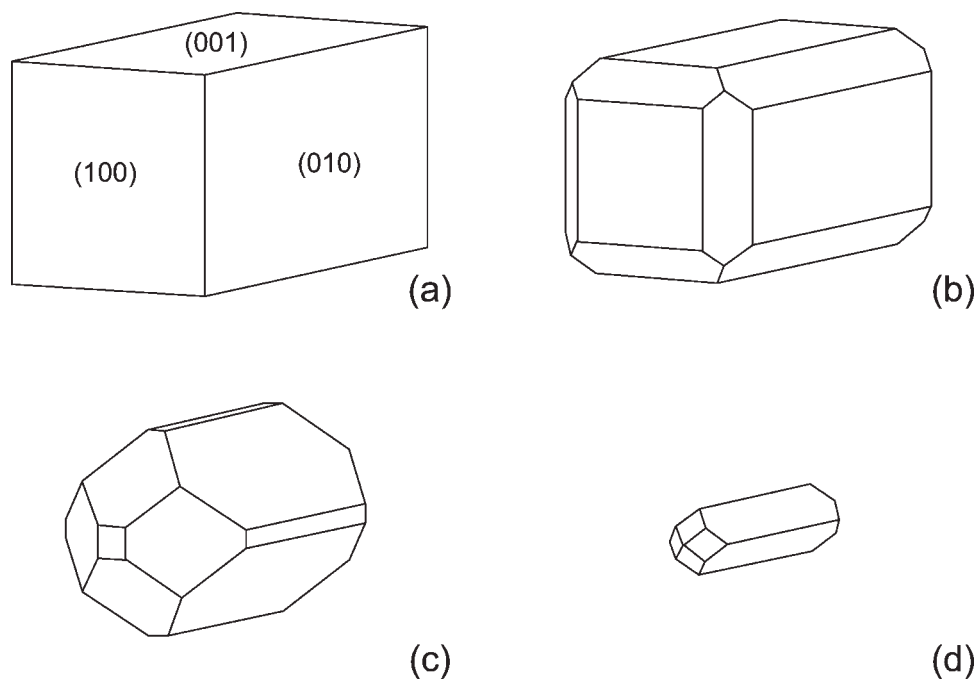


Figure 5. Dissolution shape evolution of the illustrative system under the condition, where the relative dissolution rate of stepped faces and kinked faces are approximately equal.

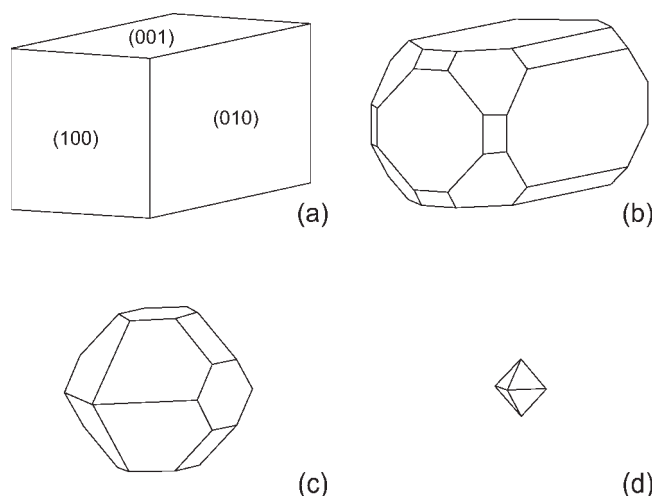


Figure 6. Dissolution shape evolution of the illustrative system under the condition where the relative dissolution rate of kinked faces is much faster than the dissolution rate of stepped faces.

We now report results of a dissolution simulation for a crystal under the conditions where kinked faces dissolve more rapidly than stepped faces. Thus, we now dissolve the stepped faces (corresponding to the $\{011\}$, $\{101\}$ and $\{110\}$ plane families) with a dissolution rate of -10 and the kinked faces (corresponding to the $\{111\}$ plane family) with a dissolution rate of -30 . The simulation is again initialized with the steady-state growth shape (Figure 6a). Figures 6b–d show the results for the dissolution evolution. Note that faces now appear at the vertices and edges as expected. Thus, the resulting dissolution shapes are more diamond-like and equant, than the more needle-like crystals in Figure 5. Again, only those sets of faces selected here are needed to describe the evolution under the condition of neglecting effects of the bulk diffusion field. None of the molecules at any of the new vertices on the dissolving crystal can be removed more rapidly than the molecules on the adjacent kinked faces.

Example succinic acid

Succinic acid is a C4 dicarboxylic acid used in the food, pharmaceutical and cosmetic industries. Its growth evolution in aqueous solution was previously reported²⁹ using a 2-D shape evolution model. However, a 2-D shape evolution is not sufficient for a dissolution study since it cannot account for the effect of new faces appearing at edges or vertices that are not in the 2-D plane of study. Our results reported below show that the crystal quickly loses its plate-like 2-D appearance during dissolution.

The relative growth and dissolution rates of succinic acid in water have been calculated using the spiral growth model. Because the growth and dissolution occurs in solution, growth and dissolution of both stepped and kinked faces will be limited by diffusion and thus approximately equal for both. Thus, their velocities are both assumed to be an order of magnitude faster than the slowest growing or dissolving face. Winn and Doherty²⁵ calculated the desired kink and edge energetics, and the resulting spiral shapes for the imple-

mentation of the spiral model. The relevant flat faces on succinic acid correspond to the $\{100\}$, $\{020\}$ and $\{011\}$ plane families which have relative growth and dissolution rates of 1.0, 4.19 and 2.98, respectively. For this study, bond chains with kink energies less than RT in energy are not considered strong bond chains, and do not count toward a face being considered flat.

First we must select the set of candidate planes. The faces in the list above correspond to the flat faces required in the list of candidate planes. The $\{1\bar{1}1\}$ family of planes does contain two bond chains, thus, it would be classified as a flat plane; however, since one of the bond chains has an energy of interaction with the solvent required to form a kink less than RT at room temperature (weak bond chain), it will act as a stepped face in dissolution. Thus, the 18 planes in the $\{1\bar{1}1\}$, $\{11\bar{1}\}$, $\{002\}$, $\{1\bar{2}0\}$, $\{120\}$ and $\{031\}$ plane families are those at the edges formed by the flat faces, and the 12 planes corresponding to the $\{102\}$, $\{10\bar{2}\}$, and $\{131\}$ plane families are the planes at the vertices formed by the adjacent flat faces. Those are also all included in the candidate set of planes. Finally, any faces on an initial condition, but not in that list are added to the superset.

The seed for the growth evolution is chosen to be the succinic acid equilibrium shape. The relative surface energy values for each of the planes are 1, 1.09, 0.48 and 1.65, respectively, for the $\{100\}$, $\{020\}$, $\{011\}$ and $\{1\bar{1}1\}$ plane families. These values were calculated by including only in-

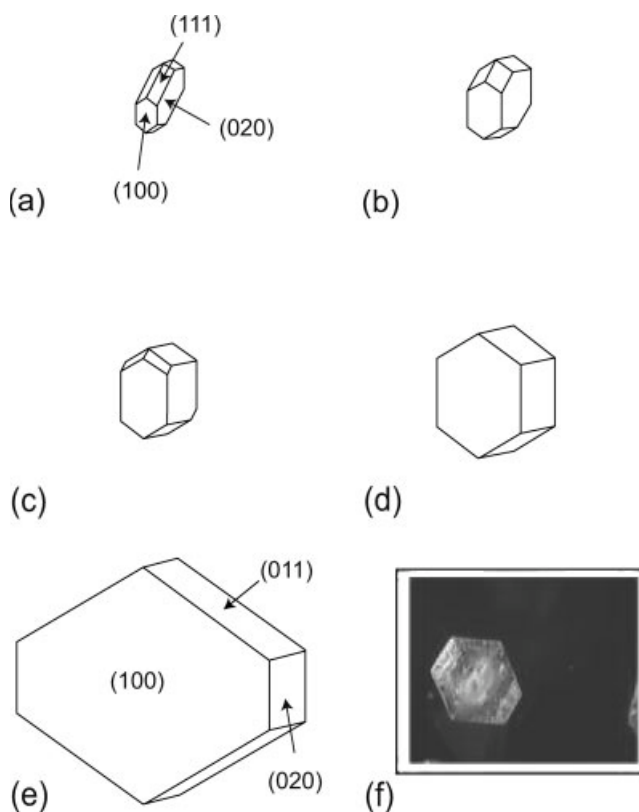


Figure 7. Shapes of succinic acid crystals growing in water from the predicted equilibrium shape (a), to its steady-state growth shape (e).

An experimentally grown crystal shape is shown in f. (Part e not to scale).

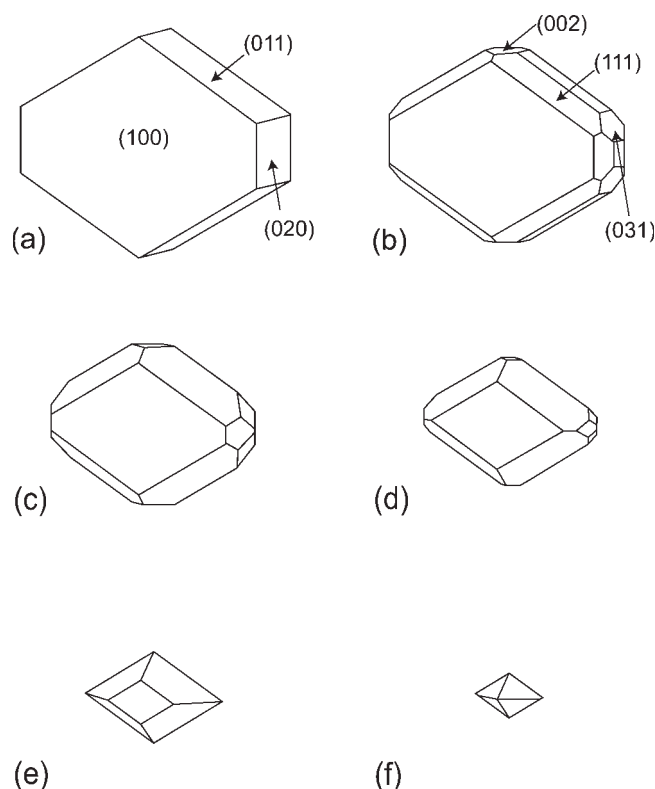


Figure 8. Shape of a succinic acid crystal dissolving in water from the steady-state growth shape.

ternal solute-solute interactions within the succinic acid crystal. The resulting equilibrium shape is shown in Figure 7a (as determined from the Wulff construction). The relative growth rate of the stepped and kinked faces are all chosen to be an order of magnitude larger ($R_i = 10$) than the slowest growing face. The predicted growth evolution of succinic acid is shown in Figure 7a–e. As Gibbs⁹ (page 325) suggested “a large crystal, thus, formed, will generally be bounded by those surfaces alone on which the deposit of new matter takes place least readily—the relative development of the different kinds of sides—will not be such as to make $\Sigma(\sigma s)$ [total surface energy] a minimum.” In other words, the shape of a grown crystal is not determined solely by thermodynamics, but rather by the relative growth rates of the crystal faces. The resulting steady-state shape is similar to experimental measurement (Figure 7f).

This steady-state growth shape is taken as the seed shape for predicting a dissolution evolution, which is shown in Figure 8a–f. Since all of the faces on this seed shape are already in the candidate face list from above, that list is sufficient for the dissolution prediction. A single new face appears at each edge, and since none of those faces that appear are flat faces only one new face is maintained at each edge. Again, the $\{111\}$ set of faces does have a second bond chain in them; however, the energy of that chain is very low ($1 \text{ kJ/mol} \ll RT$), and is for these purposes not considered to be a flat face when interfaced with the solvent. Thus, it will appear as a single face in dissolution rather than two adjacent faces appearing. Since the (virtual) planes at the vertices dissolve at approximately the same rate as the faces at the edges, no

faces appear at the vertices because the molecules at the vertices are just as easily removed from each of the edges that form the vertex.

As the eigenvalue analysis of the system of differential equations suggested, it is immediately apparent that there is a steady-state growth shape that the crystal evolves towards during growth. It is more difficult to visually determine if there is a steady-state shape that the crystal evolves toward or away from during the dissolution evolution. To this end the dimensionless perpendicular distance x_i is plotted against the warped time ξ , for several faces in both the growth evolution (Figure 9a) and the dissolution evolution (Figure 9b). The reference face for growth is $\{100\}$, and the reference face for dissolution is $\{111\}$. For the growth evolution, each of the dimensionless perpendicular distances approach the steady-state value (which is equal to the dimensionless growth rate R_i). Also note that the $\{111\}$ faces disappear in the course of the growth evolution before it can approach the steady-state value. For the dissolution evolution, as expected, the dimensionless perpendicular distance indeed evolves away from the steady-state value. Figure 9b highlights the

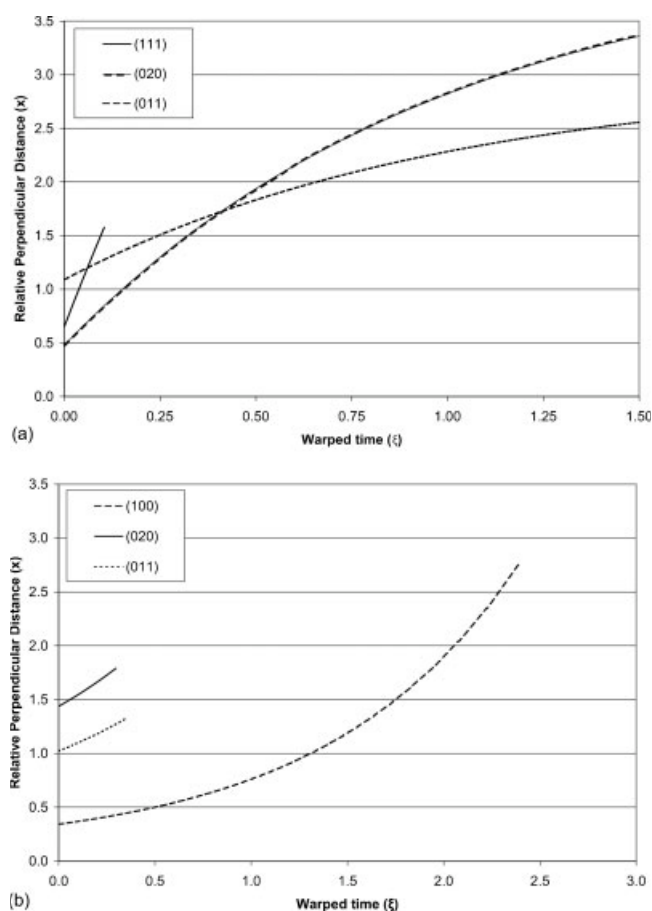


Figure 9. Relative perpendicular distance vs. warped time for selected crystal faces of succinic acid in aqueous solution.

Note that during growth (a) the relative distance approaches a steady-state value, while during dissolution (b) the relative distance moves away from the steady-state value.

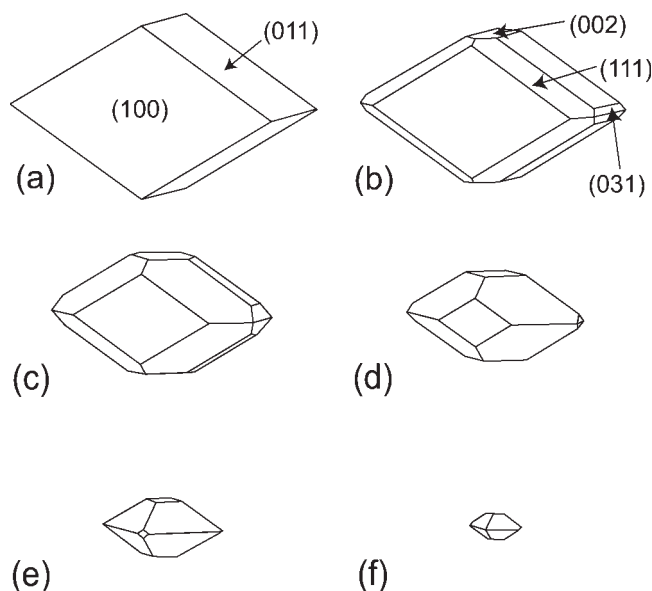


Figure 10. Shape of a succinic acid crystal dissolving in water from a diamond plate-like shape.

fact that crystal shapes evolve away from their steady-state values in dissolution.

In order to further demonstrate that there is not a unique (or stable) end-shape in dissolution, a second dissolution evolution is reported using a second initial condition for succinic acid dissolving in water. The second evolution also highlights that the attempted appearance of a flat face at an edge results not in that face appearing, but rather two others appearing. This second initial condition is shown in Figure 10a. The diamond plate-like shape is dissolved using the same dissolution rates as in the previous example. The superset of candidate faces from the previous example is also sufficient for this case. The predicted dissolution evolution is shown in Figure 10b–f. Note that two new faces appear at each end of the diamond plate since there is an additional set of flat planes ($\{002\}$) at those edges. The resulting dissolution shapes in this case are dramatically different from the dissolution shapes obtained with the previous initial condition.

Conclusions

We have developed a model that determines the shape of crystals as they dissolve or grow. The nondimensional form of the model proves that during growth, crystals evolve towards a unique steady-state; however, in dissolution, crystals evolve away from the unique steady-state. This model requires only crystallographic information as well as relative growth and dissolution rates of a set of candidate crystal faces. These values can be taken from either experiments or predictions. For organic molecular systems in solution, we have also demonstrated a methodology for predicting the relative growth and dissolution rates providing for a fully predictive methodology for the determination of dynamic crystal shape evolution for those systems.

We have quantitatively demonstrated that crystals in dissolution obtain a completely different set of crystal shapes.

Additionally, different crystal planes are exposed during dissolution than can be obtained through growth alone. In growth, since crystal shapes evolve towards their steady-state, disturbances in the system will have a negligible effect over time; however, in dissolution, any disturbance to the system will affect the shapes that are obtained. Thus, in dissolution a much higher degree of control is necessary over the process.

The use of both growth and dissolution in a cycling mode also is a potentially useful area of research for manipulating crystal shape. By utilizing the nonreversibility of the growth, and dissolution shape predictions, due to anisotropy in relative growth and dissolution rates or the systematic disappearance of crystal faces; a series of growth and dissolution steps could provide for an additional processing technique to improve crystal shapes.

Acknowledgments

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Appendix

In dissolution, the transition from the spiral to the 2-D nucleation mechanism occurs at a relatively lower-chemical potential difference, than the same transition occurs in growth. Also, during dissolution, as the undersaturation is increased etch pitting occurs. Both of these phenomenon are a result of the fact that 2-D dissolution nuclei preferentially form on dislocations, whereas growth nuclei preferentially form away from dislocations.

In order to understand this phenomenon, a Gibbs-Thompson analysis is performed for the case of forming a disc-shaped nucleus both on and away from a dislocation in both growth and dissolution. Equation 16 shows the change in free energy upon the formation of such a disc-shaped nucleus on both a flat (left) and dislocated (right) surface for growth, and Eq. 17 shows the same for dissolution

$$\Delta G = -\frac{\pi r^2 h}{V_m} \Delta\mu + 2\pi r h \gamma, \quad (16)$$

$$\Delta G = -\frac{\pi r^2 h}{V_m} \Delta\mu + 2\pi r h \gamma + h E_{disloc}$$

$$\Delta G = \frac{\pi r^2 h}{V_m} \Delta\mu + 2\pi r h \gamma, \quad (17)$$

$$\Delta G = \frac{\pi r^2 h}{V_m} \Delta\mu + 2\pi r h \gamma - h E_{disloc}$$

Here, r and h are the radius and height of the disc, V_m is the molar volume of the solute, γ is the surface energy, E_{disloc} is the strain energy per unit depth of the dislocation, and $\Delta\mu = \mu - \mu_{sat}$ is the chemical potential difference between the sol-

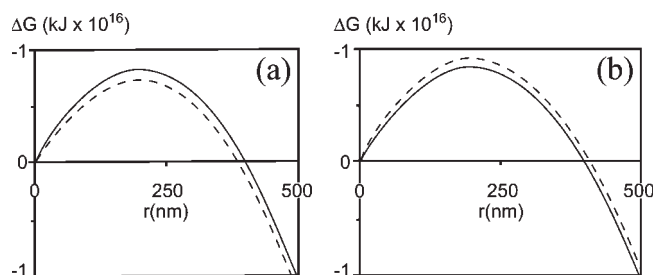


Figure A1. Gibbs-Thompson plots of a disc shaped α -glycine nucleus in water for both dissolution (a) and growth (b) for both the case including the dislocation energy (dashed line) and excluding the dislocation energy (solid line).

ute in solution and in the crystal phase ($\Delta\mu > 0$ in growth, $\Delta\mu < 0$ in dissolution). The first term in each equation represents the change in volume energy, the second term expresses the surface energy change and the third term, for the equations on a dislocation, is the strain energy associated with adding or removing an additional dislocated layer of solute. Many expressions have been proposed for the energy of the dislocation, the most widely used are those proposed by Cabrera and Levine²² and van der Hoek et al.³⁰ Plots of ΔG vs. r calculated from these nucleation equations for α -glycine in water are shown in Figure A1 using the strain function of van der Hoek et al.

In growth, the energy barrier for the formation of a nucleus is lower when it is formed away from the dislocations; however, in dissolution the barrier is lower when the nucleus is formed on the dislocation. This is because adding a layer of dislocated material in growth adds to the amount of strain energy in the system, whereas removing a layer of dislocated material from the surface relieves an additional amount of strain energy. Thus, in dissolution, nuclei will preferentially form on dislocations, while in growth they will preferentially form on terraces away from the dislocations. At higher levels of undersaturation, the 2-D nuclei are removed from the same location repetitively which results in the formation of etch pits. Additionally, since the barrier for the preferred location in dissolution is smaller than the barrier for the preferred location in growth, the transition to the 2-D nucleation mechanism occurs at a lower relative chemical potential difference in dissolution compared to growth (see Figure A1). Finally, despite these energetic preferences in growth and dissolution, the value for the critical radius is very similar for both cases. Thus, we use the critical length calculated from the traditional Gibbs-Thompson analysis for shape calculations since this does not require knowledge of the physical property value E_{disloc} .

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